APPENDIX

Properties of congruence and equivalence

Direct equivalence	$\equiv$	... §5.6
Strong congruence	$\sim$	... §5.7
Observation equivalence	$\approx$	... §7.2
Observation congruence	$\overset{c}{\approx}$	... §7.3

$B \equiv C$ implies $B \sim C$ implies $B \overset{c}{\approx} C$ implies $B \approx C$ ... Ex 5.2, Cor 7.6

Observation congruence "$\overset{c}{\approx}$" is also denoted by equality "=", though many laws (as their names indicate) hold for strong congruence "$\sim$" or even direct equivalence "$\equiv$". Except where indicated, the laws are those of Theorems 5.3 and 5.5 generalised by Theorem 5.7 .

Summation

Sum $\equiv$ (1) $B_1 + B_2 = B_2 + B_1$

(2) $B_1 + (B_2 + B_3) = (B_1 + B_2) + B_3$

(3) $B + NIL = B$

(4) $B + B = B$

Action

Act $\equiv$ $\alpha\tilde{x}.B = \alpha\tilde{y}.B\{\tilde{y}/\tilde{x}\}$

where $\tilde{y}$ is a vector of distinct variables not in B .

Composition

Com $\equiv$ Let B and C be sums of guards. Then

$B|C = \sum\{g.(B'|C);\ g.B' \text{ a summand of } B\}$
$+ \sum\{g.(B|C');\ g.C' \text{ a summand of } C\}$
$+ \sum\{\tau.(B'\{\tilde{E}/\tilde{x}\}|C');\ \alpha\tilde{x}.B' \text{ a summand of } B \text{ and } \bar{\alpha}\tilde{E}.C' \text{ a summand of } C\}$
$+ \sum\{\tau.(B'|C'\{\tilde{E}/\tilde{x}\});\ \bar{\alpha}\tilde{E}.B' \text{ a summand of } B \text{ and } \alpha\tilde{x}.C' \text{ a summand of } C\}$

provided that in the first (second) summand no free variable of C(B) is bound by g.

Com ~ (1) $B_1|B_2 = B_2|B_1$

(2) $B_1(B_2|B_3) = (B_1|B_2)|B_3$

(3) $B|NIL = B$

Restriction

Res ≡ (1) $NIL\backslash\beta = NIL$

(2) $(B_1 + B_2)\backslash\beta = B_1\backslash\beta + B_2\backslash\beta$

(3) $(g.B)\backslash\beta = \begin{cases} NIL \text{ if } \beta = name(g) \\ g.(B\backslash\beta) \text{ otherwise} \end{cases}$

Res ~ (1) $B\backslash\alpha = B$ $(B{:}L,\ \alpha \notin names(L))$

(2) $B\backslash\alpha\backslash\beta = B\backslash\beta\backslash\alpha$

(3) $(B_1|B_2)\backslash\alpha = B_1\backslash\alpha|B_2\backslash\alpha$

$(B_1{:}L_1, B_2{:}L_2,\ \alpha \notin names\ (L_1 \cap \bar{L}_2))$

Relabelling

Rel ≡ (1) $NIL[S] = NIL$

(2) $(B_1 + B_2)[S] = B_1[S] + B_2[S]$

(3) $(g.B)[S] = S(g).(B[S])$

Rel ~ (1) $B[I] = B$ $(I{:}L \to L$ the identity mapping$)$

(2) $B[S] = B[S']$ $(B{:}L$ and $S\restriction L = S'\restriction L)$

(3) $B[S][S'] = B[S' \circ S]$

(4) $B[S]\backslash\beta = B\backslash\alpha[S]$ $(\beta = name\ (S(\alpha)))$

(5) $(B_1|B_2)[S] = B_1[S]|B_2[S]$

Identifier

Ide ≡ Let $b(\tilde{x}) \Leftarrow B_b$; then

$b(\tilde{E}) = B_b\{\tilde{E}/\tilde{x}\}$

Conditional

Con ≡ (1) If true then B_1 else $B_2 = B_1$

(2) if false then B_1 else $B_2 = B_2$

Unobservable action τ

(1) $g.\tau.B = g.B$

(2) $B + \tau.B = \tau.B$

(3) $g.(B + \tau.C) + g.C = g.(B + \tau.C)$

... Theorem 7.13 (for (1)–(3))

(4) $B + \tau.(B + C) = \tau.(B + C)$ ... Cor. 7.14

<u>Observation Equivalence</u>

(1) $B \approx \tau.B$ ... Prop. 7.1

(2) $\approx$ is preserved by all operations except + ... Theorem 7.3

(3) $B \approx C$ implies $B = C$ when B,C stable ... Prop. 7.11

(4) $B \approx C$ implies $g.B = g.C$ ... Prop. 7.12

<u>Expansion</u> ... Theorem 5.8

Let $B = (B_1|\ldots|B_m)\backslash A$, where each B_i is a sum of guards. Then

$$B = \sum\{g.((B_1|\ldots|B_i'|\ldots|B_m)\backslash A);\ g.B_i' \text{ a summand of } B_i,\ \text{name}(g) \notin A\}$$
$$+ \sum\{\tau.((B_1|\ldots|B_i'\{\tilde{E}/\tilde{x}\}|\ldots|B_j'|\ldots|B_m)\backslash A);\ \alpha\tilde{x}.B_i' \text{ a summand of } B_i,\ \bar{\alpha}\tilde{E}.B_j' \text{ a summand of } B_j,\ i \neq j\}$$

provided that in the first term no free variable in $B_k (k \neq i)$ is bound by g.

References

(In these references, LNCSn stands for Lecture Notes in Computer Science, Vol n, Springer Verlag.)

[Bri1] P. Brinch Hansen, Operating Systems Principles, Prentice Hall, 1973.

[Bri2] P. Brinch Hansen,"Distributed processes; a concurrent programming concept", Comm. ACM 21, 11, 1978.

[CaH] R. Campbell and A. Habermann, "The specification of process synchronization by Path Expressions", LNCS 16, 1974.

[Car] L. Cardelli, "Analog Processes", To appear in Proc 9th MFCS, Poland, 1980.

[CoH] F. Commoner and A. Holt, "Marked directed graphs", JCSS 5, 1971.

[DFL] J. Dennis, J. Fosseen and J. Linderman, "Data flow schemas", LNCS 5, 1974.

[Dij] E. Dijkstra, "Guarded commands, nondeterminacy and formal derivation of programs", Comm. ACM 18, 8, 1975.

[EBJ] P. van Emde Boas and T. Janssen, "The impact of Frege's principle of compositionality for the semantics of programming and natural languages", Report 79-07, Dept. of Mathematics, University of Amsterdam, 1979.

[GLT] H.J. Genrich, K. Lautenbach and P.S. Thiagarajan, "An overview of Net Theory", Proc. Advanced Course on General Net Theory of Processes and Systems, to appear in LNCS, 1980.

[HAL] C. Hewitt, G. Attardi and H.Liebermann, "Specifying and proving properties of guardians for distributed systems", LNCS 70, 1979.

[HM] M. Hennessy and R. Milner, "On observing nondeterminism and concurrency", to be presented at 8th ICALP at Amsterdam, and appear in LNCS, 1980.

[Hoa1] C.A.R. Hoare, "Towards a theory of parallel programming", in Operating Systems Techniques, Academic Press, 1972.

[Hoa2] C.A.R. Hoare, "Monitors: an operating system structuring concept", Comm. ACM 17, 10, 1974.

[Hoa3] C.A.R. Hoare, "Communicating Sequential Processes", Comm. ACM 21, 8, 1978.

[HP1] M. Hennessy and G. Plotkin, "Full abstraction for a simple parallel programming language", Proc 8th MFCS, Czechoslovakia, LNCS 74, 1979.

[HP2] M. Hennessy and G. Plotkin, "A term model for CCS", to appear in Proc 9th MFCS, Poland, 1980.

[Hue] G. Huet, "Confluent reductions: abstract properties and applications to term-rewriting systems", Report No. 250, IRIA Laboria, Paris 1977.

[Kel] R. Keller, "A fundamental theorem of asynchronous parallel computation", Parallel Processing, ed. T.Y. Feng, Springer, 1975.

[KMQ] G. Kahn and D. MacQueen, "Coroutines and networks of parallel processes", Proc. IFIP Congress, North Holland, 1977.

[Kun] H.T. Kung, "Synchronized and asynchronous algorithms" in Algorithms and Complexity, ed J.F. Traub, Academic Press, 1976.

[Mil1] R. Milner, "Processes; a mathematical model of computing agents", Proc Logic Colloquium '73, ed. Rose and Shepherdson, North Holland, 1973.

[Mil2] R. Milner, "Flowgraphs and flow algebras", J. ACM 26, 4, 1979.

[Mil3] R. Milner, "Synthesis of communicating behaviour", Proc 7th MFCS, Poland, LNCS 64, 1978.

[Mil4] R. Milner, "Algebras for communicating systems", Report CSR-25-78, Computer Science Dept., Edinburgh University, 1978.

[Mil5] R. Milner, "An algebraic theory of synchronization", LNCS 67, 1979.

[Mln] G. Milne, "A mathematical model of concurrent computation", Ph.D. Thesis, Computer Science Dept, University of Edinburgh, 1978.

[MM] G. Milne and R. Milner, "Concurrent processes and their syntax", J. ACM, 26, 2, 1979.

[Mos] P. Mosses, "SIS, Semantic Implementation System", DAIMI Report MD-33, Aarhus University, 1979.

[MQ] D. MacQueen, "Models for distributed computing", Report No. 351, IRIA-Laboria, Paris, 1979.

[Mül] T. Müldner, "On synchronizing tools for parallel programs", Report 357, Inst. of Computer Science, Polish Academy of Science, Warsaw, 1979.

[MWW] A Maggiolo-Schettini, H. Wedde and J. Winkowski, "Modelling a Solution for a control problem in distributed systems by restrictions, LNCS 70, 1979.

[Pet] C.A. Petri, "Introduction to General Net Theory", Proc. Advanced Course on General Net Theory of Processes and Systems, to appear in LNCS, 1980.

[Plo1] G. Plotkin, "A powerdomain construction", SIAM J. Comp 5, 1976.

[Plo2] G. Plotkin, "LCF considered as a programming language", TCS 5, 3, 1977.

[Pnu1] A. Pnueli, "The temporal logic of programs", 19th Annual Symp. on Foundations of Computer Science, Providence R.I., 1977.

[Pnu2] A. Pnueli, "The temporal semantics of concurrent programs", LNCS 70, 1979.

[Ros] B. Rosen, "Tree manipulation systems and Church-Rosser Theorems", J. ACM 20, 1, 1973.

[Sch] J. Schwarz, "Distributed synchronization of processor communication", Internal Report, Dept. of Artificial Intelligence, University of Edinburgh, 1978.

[Smy] M. Smyth, "Powerdomains", JCSS 16, 1978.

[SS] D. Scott and C. Strachey, "Towards a mathematical semantics for computer languages", Proc. Symp. on Computers and Automata, Microwave Res. Inst. Symposia Series, Vol 21, Polytechnic Inst. of Brooklyn, 1972.

[Wad] W. Wadge, "An extensional treatment of dataflow deadlock", LNCS 70, 1979.

[Wir] N. Wirth, "MODULA: A language for modular multiprogramming", Report 18, ETH Zurich, 1976.